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Enhancing Malaysia's human capital: A study of students' preference and usage level of graphs in solving applied derivative problems

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Abstract

Developing countries such as Malaysia need to cultivate and harness the problem solving skills of their potential workforce in order to be economically competitive. Cartesian graphs are known to be effective tools for solving derivative problems. Although 'sketching a graph' is advocated as a useful strategy, generating an appropriate graph may pose difficulties and consequently cause the students to reluctantly employ them. This study examined the Malaysian students' preference and usage levels of graphs in solving applied derivative problems. A 16-item Likert-scale questionnaire consisting of four categories and two tasks on the application of derivative were distributed to 194 pre-university students in Selangor. The results showed a negative relationship between the students' actual preference method and their usage level of graphs in solving derivative problems. This implied that teachers should be encouraged to motivate students to practically utilize graphs in their learning of mathematical concepts by increasing the use of graphs creatively. The design of the instructional materials and the questioning and examination tasks should gear towards the promoting of information displayed graphically. It was also recommended that the variations in the types and functions of graphs be highlighted so as to seek students' skills and ability to sketch graphs and to read and interpret graphs efficiently and effectively.

Keywords: applied derivative problems, graphs, instructional materials, mathematics education, problem solving skills, students' preference

Introduction

The past decade has seen calls been made to search for new different perspectives of comprehending mathematical concepts that should differ from the traditional method of teaching and learning. The new perspective identified the urge that knowledge is structured to relate to real life situations. Educators, mathematicians and researchers observe that traditional teaching stressed on procedural knowledge rather than conceptual thinking (Macini & Gagnon, 2006; Parmjit & White, 2006) and activities which include sequence of cognitive processes: understanding sentence-accompanied problem, selecting the correct data and plan for solution, solving the problem and reasoning the answer (Polya, 1945). Educators should introduce techniques that strategize to embed effective mathematical solving tools into real life situations and at the same time merge the use of procedural knowledge and conceptual knowledge that will increase the value of understanding mathematics among students (Nor'ain & Mohan, 2016; Guler & Ciltas, 2011). Brown (1983) proposed that, in promoting a particular strategy for students to solve mathematical problem, it is important to provide skills and knowledge on the effectiveness of the strategy. Of the various techniques available, Manalo & Uesaka (2011) promoted that the use of diagrams to be the best approach in solving mathematical problems and subsequentlyemphasized that the use of diagrams to

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depend largely on how students perceive the efficiency of employing diagrams and the skills in constructing the diagrams (Uesaka, Manalo & Ichikawa, 2010). The approach to enhance the strategy used in solving mathematical problems through the use of graphs, be taken as one type of diagrams, or representations in general, may prove useful not only graphs are tools for solving problems but empirical evidence by research that has explored graphs as tools for communications (Friel, Curcio & Bright, 2001; Leung & Chan, 2004; Sharma, 2013). The effectiveness of graphs as tools for communication of mathematical concepts and ideas has been demonstrated in some earlier studies. For example, Leung and Chan (2004) reported that graphs had help students to convey their understanding in identifying patterns of graphs of functions. This suggest that providing the students with the opportunity to use graphs to reason could promote their opting for graphs when strategizing for solutions without the necessity for teacher to encourage the move.

The importance of developing skills in graph use

Graphs are one of the common mathematical tools to convey information. They are used in managing, communicating and analysing information. Within school contexts, they are used in textbooks and examinations or other educational print and electronic media (Shah & Hoeffner, 2002). The graph of functions representing various mathematical situations is of utmost important for students to comprehend mathematical concepts. The effectiveness of sketching graphs has been empirically demonstrated (Friel et al., 2001; Leung & Chan, 2004; Lowrie, 2005; Sedig, 2004). Researchers and educators working in the area of graph usage have discussed not only on the beneficial effects that they imposed but also the mechanisms and means by which they support the results in solving problems (Uesaka et al., 2011). Successful use and applications of graphs as representations and visual information depend upon skilful matching of the particular function with the real life problem it is standing for. Western and developed countries such as United States of America and Australia emphasized on the skills required for pupils to sketch and interpret graphs within schools contexts (NCTM, 2000; SACE, 2014) to be included in their primary school curriculum. NCTM (2000) stressed that students must be able to use or refer to different representations such as graph to enhance their learning methods. Visual representations, Cartesian graphsin this study, are considered the most efficient visualization strategy in the learning of mathematical concepts and applications into problem solving especially those related to the daily situations.

Students' reluctance to use graphs

Although Stern, Aprea and Ebner (2003) claimed that self-constructed graphs are promising tools for understanding mathematical concepts and solving mathematical problems, researchers and educators did encounter problems related to students' use of graphs. This includes their choice of methods that prone to the verbal or analytical method and their unable to sketch the correct graphs for the problems attempted (Uesaka et al., 2010, 2011). When students opted not to sketch graphs to work through the solution, they are actually oblivious of the graphs' efficacy in assisting the solving process particularly when neither diagrams nor hints were accompanying the sentential problems. Uesaka, Manalo& Ichikawa, (2007)had also identified that students reluctance to use graphs resulted from their perceptions on the efficiently of the graphs as representations and solving tools and the difficulties in using the graphs. In another study that followed, they validated that the skills to sketching graphs together with their positive perception on the efficiency of graphs contribute to the students' choice of using graphs to help them in the solving of problems. Students must be taught and encourage to draw graphs as tools that help them to solve and at the same time to communicate ideas and understanding of the mathematical concepts (Novick, 2006). Research conducted concerning students learning approach suggested that the higher the cognitive effort required to perform any method to solutions, the lower the chances of students to opt for the particular approach (Muruyama, 2003). Sketching graphs as a solution method is considered as one of the learning approach. If a student requires more cognitive or mental effort in order to sketch related graphs for the problem situation then the less likely for graphs to be considered as a choice.

Uesaka & Manalo (2011) elaborated stages for students to undertake starting from the process of reading the words through the solution of the problems using graphs. Students need to visualize the problem situation in terms of relationships among the components in the problem. They will then transform the relationships into graphs by carefully extracting only those relevant information about the problem and discarding those irrelevancies that usually or may be part of the system of the problem presentation. The crucial cognitive demand required in the course of action lies in the transforming the relationships to the graphical forms. This refers to the implementation restraint needed to execute the tasks (Miller & Cohen, 2001). Students would need to transform the relationships into graphs which, although is very effective in portraying the quantitative data and relationships, they are in fact very abstract and artificial since they do not describe 'systems in real spaces' (Larkin & Simon, 1987, p.93). Action such as identifying concepts and connections and making decision and evaluation of the problem situations must be operated at a higher degree in making sure the effectiveness of their representations and used in graphs. The cognitive demand involved could therefore be quite extreme and eminent and may also be unaffordable to some students.

Purpose of the study

The use of graph in the problem solving process may not always produce effective results. In some situations, the perception students have on the difficulties faced in using and comprehending graphs may lead to incorrect answers (Lean & Clement, 1981; Presmeg, 1992). These may also be influenced by how and the extent teachers employ the use of graphs in classroom environment. Students tend to adopt their teachers' method of solving solution, either using graphs or verbal solutions. The present study aimed at determining whether there is a relationship between the students'usage level on graphs use in the teaching and learning of mathematics and their preference methods in solving sentential mathematical problems. The study is specifically aimed to answer the following research questions:

1. What is the students' preference method when solving derivative problems?
2. What are the students' usage level graphs in solving mathematical problems?
3. Is there any correlation between the students' preference method and their usage level of graphs in solving mathematical problems?

Method

Participants of the study were 194 students enrolled in the South Australian Matriculation Programme at a college in Selangor, Malaysia. Their ages ranged between 18 to 19 years old. They were selected students who excelled in the Malaysian Certificate of Education (SPM) examination, the main Malaysian national examination, that had been offered scholarship by various authorities and bodies to further their studies abroad in various discipline such as medicine, engineering, sciences and commerce. At the time of the study, the students had already finished the topic Differential Calculus of the South Australian Certificate of Education's syllabus. They are equipped with graphic calculators to assist them in understanding the concepts and solving the problems.

In the present study, the first part of the study dealt with a self-constructed instrument, the Mathematical Visuality Test. The students were provided with two sentential tasks which do not contain any graph, figure or diagram. In responding to the tasks, the students may choose any of their preferred method either through the manipulation of algebraic expressions or to use graphs or any other figure or diagrams to illustrate and explain the solutions.

Task 1

The number of unemployed people u at time t was studied over a period of time. At the start of this period, the number of unemployed was 800 000. Throughout the period, $\frac{du}{dt} < 0$ *and* $\frac{d^2u}{d^2t} > 0$*. Describe the number of unemployed people over time.*

In Task 1, students are expected to recall the relationship between functions and their first and second derivatives, realize that the initial number of unemployed people was 800 000, the number of unemployed is decreasing due to the first derivative of the unemployed function, *u*, is less than zero (negative) and is actually decreasing with increasing rate due to the fact that the second derivative of the unemployed function, *u*, is more than zero(positive). Students may 'describe' the number of unemployed people using sentences interpreting the symbols $\frac{du}{dt} < 0$ and $\frac{d^2u}{d^2t} > 0$ or they may opt to sketch graphs to show how unemployed people changes over time.

Task 2

The number $A(t)$ *of students logged onto an educational website at any time t, over a five-hour period is approximated by the formula* $A(t) = 175 + 18t^2 - t^4$, $0 \le t \le 5$ *. Find:*

(a) the rate of change of the number of students logged onto the website after 2 hours
(b) the interval of time when the number of students logged onto the website is increasing.
(c) the interval of time when the rate of change of the number of students logged onto the website is increasing.

In Task 2, students are expected to make use of their graphic calculator to sketch the graphs of the students logged onto the educational site and consequently trace all answers (for all parts (a), (b) and (c)) from it. The rate of change of the number of students logged onto the website after 2 hours can be solved by using the derivative function key in the graphic calculator while the interval of time when the number of students logged onto the website is increasing can be traced by either from the graph of the students logged onto the educational site or by sketching the derivative function and trace the interval when the graph of the derivative function is above the *x*-axis. The interval of time when the rate of change of the number of students loggedonto the website is increasing can be found by referring to the graph of the derivative function or further sketching the graph of the second derivative of the number of students logged onto the website.

Both tasks are on the application of derivatives onto real life situations. Derivatives are customarily associated to graphs as visual representations although they are less common use in the Malaysian system of education and class environment. Most of the problems either basic concepts of differentiation or their applications on real life problems are carried out using a mixture of 'algebraic and analytical arguments' (Bardelle, 2010: p. 254). The choice of domain contents and technique of questioning is aimed at finding students 'prompt' and preference method in solving application problems. Students have the choices to answer them using the algebraic manipulation or they may sketch graphs to 'see' and elaborate the situation or to seek for solutions. Students should be very well-versed with graphic calculator since they are trained to make use of it during classroom activities.

The second part proceeded with a questionnaire of 16-item Likert-scale Visual Representation Usage Level which was developed by Uesaka, Manalo and Ichikawa (2007) with the aim to ascertain the students' use of diagrams in solving mathematical problems was adapted to identify students' perception on the graphs use in teaching and learning. Some adjustments were made to the questionnaire to suit the students' understanding of the terminology. Firstly, before the distribution of the questionnaire, the students were also consulted on their understanding of the word 'image' which some admitted that the word brings too vague or too broad meaning. Secondly, since the study focused on the use of graphs, the word 'image' in the original questionnaire was change to the word 'graph'. Thirdly, one of the items was removed due to the students' opinion that it is asking for the same idea as the other item. The two words, 'difficulty' and 'troublesome' triggered the same meaning to the students, and therefore be taken as repetitive. The questionnaire recorded four different measurements; 1) the students' preference levels on using graphs in their daily learning behaviour, 2) the students' view on the usefulness of graphs in solving mathematical problems, 3) the students' perception on the difficulty of the use of graphs in solving mathematical problems and 4) the students' perception on teacher's behaviours in using graphs in solving mathematical problems.

Findings and comments

What is the students' preference method when solving derivative problems?

In searching for the students' preference in solving the derivative problems, a rubric was prepared which consisted of seven categories : '*correct graph with correct solution'* (CGCS) where student sketched correct graph representing the situation and managed to arrive to the correct solution using the information read off from the graph, '*correct graph with incorrect solution*' (CGIS)where student sketched the correct graph representing the situation but were unable to read off the correct information from the graph leading to incorrect solution, '*incorrect graph with correct solution'*(IGCS) where students sketched incorrect graph to represent the situation but managed to obtain the correct solution based on their incorrect graph, '*incorrect graph with incorrect solution'*(IGIS) where student sketched incorrect graph and consequently arrived to incorrect solution, '*no graph with correct solution'*(NGCS) where student did not sketch any graph but used the algebraic manipulations to arrive to the correct solution, '*no graph with incorrect solution'*(NGIS) where student did not sketch any graph but used the algebraic manipulations to arrive to the incorrect solution and '*no answer or not attempted'*(NA) where made no attempt on the question. Students' solutions were grouped into respective categories and frequencies and percentages were calculated for each.

Table 1. Frequencies and percentages of the responses of the students for the Mathematical Visuality Test

Type of response	Task			
	1	2(a)	2(b)	2(c)
	f(%)	f(%)	f(%)	f(%)
CGCS	23(11.86)	0(0.00)	0(0.00)	0(0.00)
CGIS	1(0.52)	0(0.00)	0(0.00)	0(0.00)
IGCS	21(10.82)	0(0.00)	0(0.00)	0(0.00)
IGIS	13(6.70)	0(0.00)	0(0.00)	0(0.00)
NGCS	100(51.55)	162(83.51)	152(78.35)	87(44.85)
NGIS	21(10.82)	28(14.43)	28(14.43)	93(47.94)
NA	15(7.73)	4(2.06)	14(7.22)	14(7.22)
Total	194(100.00)	194(100.00)	194(100.00)	194(100.00)

From the analysis in Table 1, it can be seen that, for Task 1, more than half (62.37%) of the students opted for algebraic method to solve the problem where 51.55% managed to arrived to the correct description of the unemployed people over time. Only about 12.38% obtained the correct graphs while some 17.52% of the students produced incorrect or incomplete graphs. Figure 1 illustrates the sample of students' work who managed to describe the situation in sentences while Figure 2 shows an example of students sketching an incorrect graph to represent the situation.

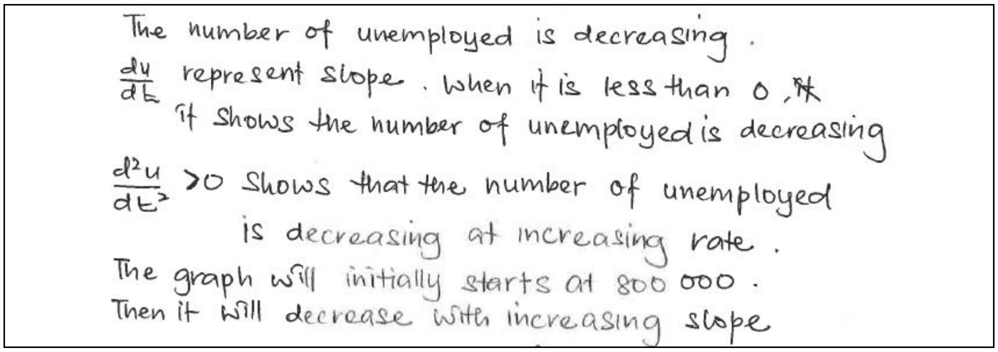


Figure 1. *An example of students' work with 'no graph correct solution' (NGCS)*

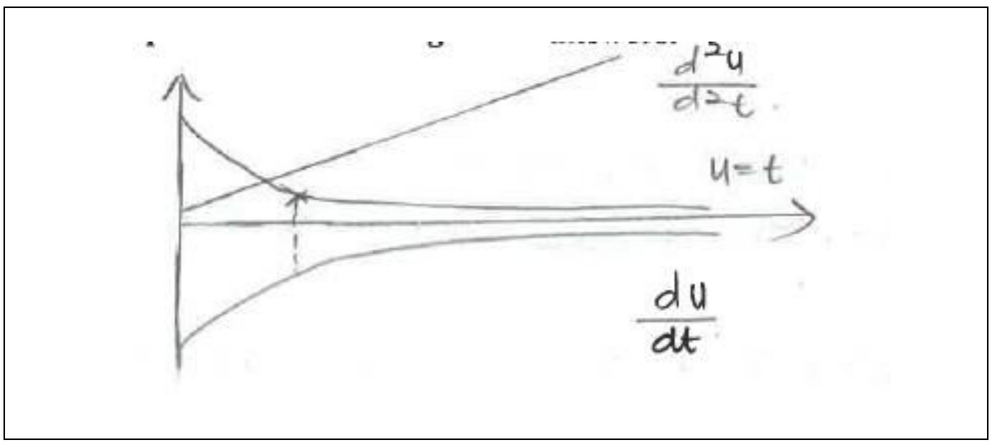
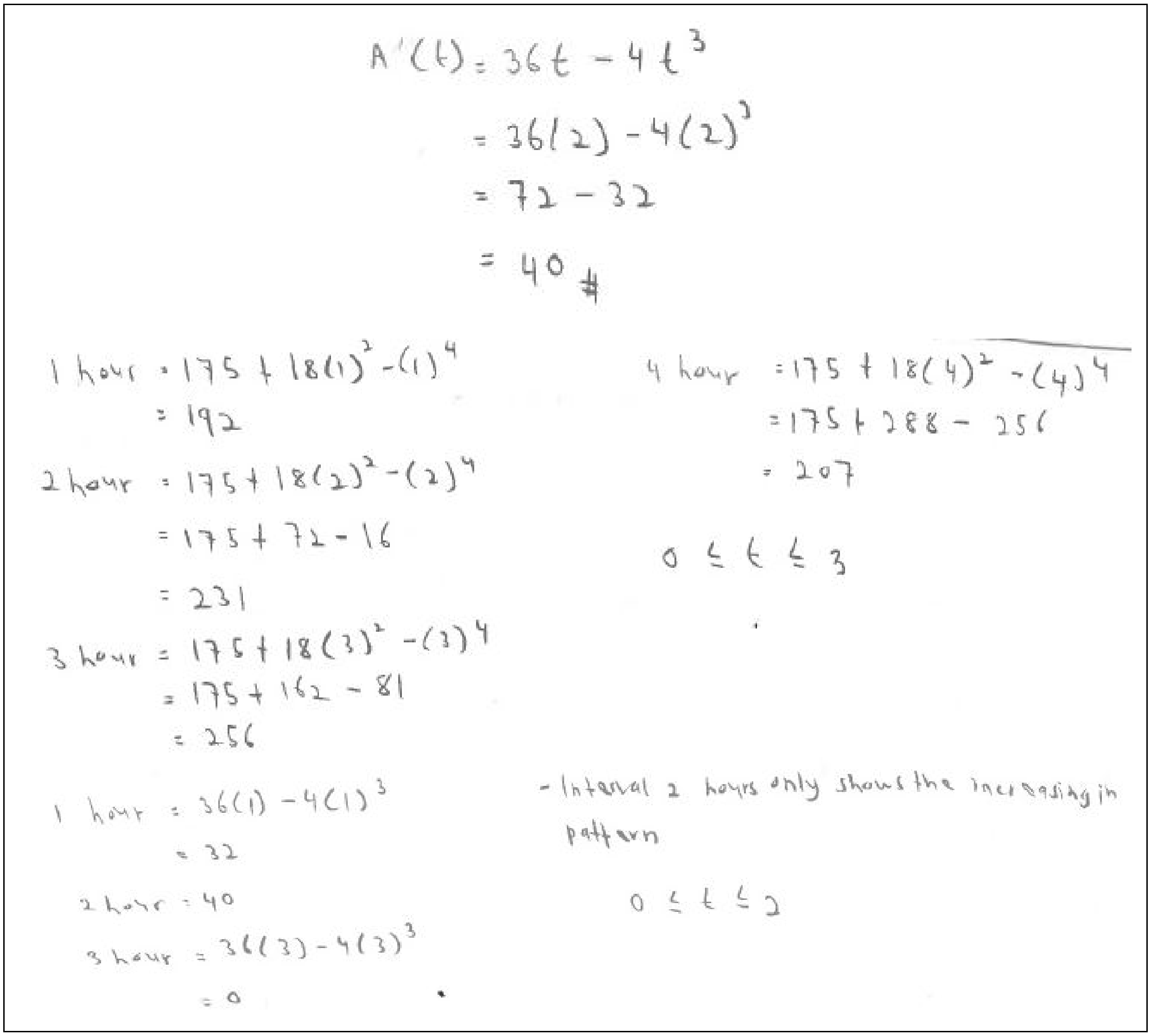


Figure 2. *An example of students' work with 'incorrect graph incorrect solution' (IGIS)*

On the other hand, analysis of Task 2 showed that students were definitely reluctant to use graphs in order to solve the derivative problems. Surprisingly, none of them used graph to search for the solutions since, as mentioned before, they are equipped with graphic calculator to help them finding the solutions. More than three quarter of the students (83.51% for Task 2(a) and 78.35% for Task 2(b)) solved the problems algebraically and managed to come out with the correct solution and 44.85% of the students were in the same category for Task 2(c) which required more cognitive process of the students. Only a small portion of less than 10% of the students did not make any attempt to try solving the problems. These percentages show that students are more 'comfortable' to solve derivative problems algebraically as compared to using graphs. The task may looked very familiar and hence automatically opted to the mathematical procedure to differentiate functions and finding the value of the independent variable, usually the '*x*' ('*t'* in this study) by equating the first or second derivative to zero in solving for the rate of change or the interval of increasing rate of change. The sample of students 'algebraic work' for Task 2 are as shown in Figure 2. The findings of the study are compatible with the study carried out by Uesakaet al (2007) on Japanese students and by Cheng (2002) and Mayer (2003).


A'(t) = 36t − 4t³
= 36(2) − 4(2)³
= 72 − 32
= 40 #

1 hour = 175 + 18(1)² − (1)⁴
= 192
2 hour = 175 + 18(2)² − (2)⁴
= 175 + 72 − 16
= 231
3 hour = 175 + 18(3)² − (3)⁴
= 175 + 162 − 81
= 256

4 hour = 175 + 18(4)² − (4)⁴
= 175 + 288 − 256
= 207

0 ≤ t ≤ 3

1 hour = 36(1) − 4(1)³
= 32
2 hour = 40
3 hour = 36(3) − 4(3)³
= 0

- Interval 2 hours only shows the increasing in pattern

0 ≤ t ≤ 2

Figure 3. *Examples of student's work for Task 2*

What are the students' usage level graphs in solving mathematical problems?

Table 2. The students' preference levels on using graphs in their daily learning behaviour

	Item	Not at all - Slightly f (%)	Moderately f (%)	Very much - Definitely f (%)
1	Do you usually use graphs in solving derivative problems?	21(10.82)	92(47.43)	81(41.75)
2	Do you try to use the kinds of graphs shown by your teacher to solve other similar derivative problems?	23(11.85)	60(30.93)	111(57.22)
3	Do you try to use the kinds of graphs shown in your textbooks to solve other similar derivative problems?	31(15.98)	58(29.90)	105(54.12)

	Item	Not at all - Slightly f (%)	Moderately f (%)	Very much - Definitely f (%)
4	Do you pay attention to the use of graphs for solving derivative word problems that your teacher shows on the board during class?	6(3.09)	25(12.89)	163(84.02)
5	Do you try to copy the way your teacher uses graphs to solve derivative words problems?	21(10.82)	68(35.06)	105(54.12)

In Table 2, the students' preference levels on using graphs in their daily learning behaviour and the responses given to the Item 1-5 of the Likert type questions were analysed. Based on the analysis, the percentage of students who answered that they 'Very much - Definitely' use graphs in solving derivative problems were 41.75%. The percentage of students who gave similar responds to whether they try to use the graphs showed by their teacher or in the textbooks were 57.22% and 54.12% respectively. 84.02% and 54.12% of the students responded 'Very much - Definitely' that they pay attention to the graphs used by their teachers and try to copy the way their teacher used graphs respectively. These percentages show that the students have positive preference on the use of graphs in solving mathematical problems. The responses also show that the use of graphs by teachers do influence the students' preference levels in using graphs for the solving mathematical problems.

Table 3. The students' view on the usefulness of graphs in solving derivative problems

	Item	Not at all - Slightly f (%)	Moderately f (%)	Very much - Definitely f (%)
6	Do you think the use of graphs is helpful in efficiently solving derivative word problems?	4(2.06)	39(20.10)	151(77.84)
7	Do you think it is good to use graphs in solving derivative word problems?	12(6.18)	45(23.20)	137(70.62)
8	Do you think the use of graphs helps you figure out how to solve derivative word problems?	4(2.06)	52(26.80)	138(71.14)

The responses given to Question 6-8 of the questionnaire and the students' view on the usefulness of graphs in solving mathematical word problems were analysed as in Table 3. Majorities of the students (77.84%) say that they 'Very much - Definitely' think that graphs is a help, 70.62% of the students said that it is good to use graphs and 71.14% of the students admitted that the use of graphs help them to figure out how to solve mathematical problems. These percentages show that the students have strong positive views that graphs are very useful in assisting them to solve derivative word problems and consequently may lead to successfully understanding mathematical concepts (Friel et al., 2001; Lowrie & Diezmann, 2011).

Table 4. The students' perception on the difficulty of the use of graphs in solving derivative problems

	Item	Not at all - Slightly f (%)	Moderately f (%)	Very much - Definitely f (%)
9	In general, do you know how to construct graphs for solving derivative word problems?	20(10.31)	90(46.39)	84(43.30)
10	How easy is it for you to use graphs in solving derivative word problems?	23(11.85)	95(48.97)	76(39.18)
11	Do you know what kinds of graphs are helpful in solving different kinds of derivative word problems?	27(13.91)	107(55.16)	60(30.93)

Table 4 present the analysis on the students' perception on the difficulty in using graphs to solve derivative problems. Almost half (46.39%) of the students said that they 'Moderately' knew how to construct graphs while another 43.30% of the students were 'Very much - Definitely' knew how to construct graphs to solve derivative problems. A bigger majority (86.09%) of the students said that they at least 'Moderately' know the different kind of graphs for particular type of derivative problems while 88.15% of the students were at least 'Moderately' find it is easy to use graphs as a method of solving derivative problems. These show that the students are not facing difficulties in understanding graphs and are able to construct graphs although a small portion of them may faced some difficulties in doing so. Students realized the need to sketch graphs in helping them to solve problems or in explaining concepts.

Table 5. The students' perception on teacher's behaviours in using graphs in solving derivative problems

	Item	Not at all - Slightly f (%)	Moderately f (%)	Very much - Definitely f (%)
12	Do your mathematics teachers use graphs to explain how to solve derivative word problems?	4(2.06)	41(21.13)	149(76.81)
13	Do you think your mathematics teachers use graphs to efficiently solve derivative word problems?	8(4.12)	31(15.98)	155(79.90)
14	Do the graphs that your teachers use to show how to solve derivative word problems help you to understand how those problems can be solved?	2(1.03)	49(25.26)	143(73.71)
15	Are you told or encouraged by your mathematic teachers to use graphs in solving derivative word problems?	17(8.76)	47(24.23)	130(67.01)
16	Do your mathematics teachers teach your class how to use graphs in solving derivative word problems?	12(6.18)	45(23.20)	137(70.62)

In Table 5, the students' perception on their teachers' behaviour on the graph usage are analysed. In Item 12, 76.81% of the students were 'Very much - Definitely' agreed that their teachers make use of graphs in the solving of derivative problems and 79.90% of the students thinks that their teacher had use graphs to efficiently solve the derivative problems respectively. In Item 14, at least 89% of the students were 'Moderately' confirmed that the graphs their teacher use to show on how to solve derivative problems had actually help them in understanding the solving process. 67.01% of the students were 'Very much - Definitely' agreed as being told or encouraged by their teachers to use graphs in solving derivative problems and 70.62% of the students were in the same categories agreed that their teachers actually taught them how to use graphs to solve derivative problems. These percentages show that the teachers' usage of graphs in solving mathematical problems influence the students' methods of solving derivative problems and support the students understanding and consequently increase their motivation and success in problem solving.

Is there any correlation between the students' preference and their actual use of graphs in solving mathematical problems?

The last part of the study found that there is a negative correlation between the students' actual preference on method employed to solve derivative application problems and their usage level of graphs in the teaching and learning of mathematics. The actual behaviour of the students can be determined through their worked solutions while the responses they gave in the questionnaire may due to wanting to show 'positive' responses although they were advised to provide their true perceptions. At least 40% and 30% of the students admitted that they were 'Very much - Definitely' try to use graphs in solving derivative problems and perceived that the use of graphs as not difficult respectively. More than 70% of the students were 'Very much - Definitely' realized the usefulness of graphs in helping them to solve derivative problems while at least 67% of the students agreed that their teacher's usage of graphs in the teaching of derivative actually motivate and encourage them to employ the same technique. On the overall, more than 84% of the students, in all items, are at least 'Moderately' agreed with the positive usage of graphs in dealing with derivative problems. Nevertheless, in analysing their worked solutions, the opposite is exposed. The results of students' works on Task 1 showed that less than 30% of the students employed graphs in their solution process while only about 12.38% managed to sketch the correct graphs for the situation and 17.52% of the students actually drew incorrect graphs. Similarly for Task 2, more than half of the students opted not to use graphs to look for the required solutions. They failed to realized that solutions to all three parts can be easily obtained from sketching the graphs of $y=A(t)$, the number of students logged onto the website. At least 92.78% of the students chose to work the solution algebraically and the majority of them managed to arrive to the correct solutions.

Conclusion and implications

The main finding of the study shows that there is a negative relationship between students' perception on using graphs and their actual use of graphs in solving derivative related problems not accompanied by any graphs.When the questionnaires were analyzed, the students were very positive about their own capabilities in using and constructing the graphs including the graphs used and taught by their teachers in the classroom. Unfortunately,the opposite was discovered from their worked solutions. They did not regard graphs as the main solutions for derivatives problems. They in fact were very favourable to the 'imitative reasoning' which Lithner (2008) defined as reasoning through the memorizing of the procedures and algorithms instead of visual or graph reasoning (Shah &Hoffner, 2010). Be that as it may, the students may still get through the hurdles of examinations and consequently proceed to their tertiary education even with such limited conceptual understanding.

The results of the study have implications for both the teaching and assessment of graphs within the derivatives calculus curriculum . Since the tasks were set very similar to classroom practice and examination testing instruments it would therefore be vital to consider the results of the study from teaching and assessment perspectives. Teachers should be encouraged to make students not only believe and perceive but also to motivate students to practically utilize graphs in their learning of mathematical concepts by increasing the use of graphs creatively. The design of the instructional materials and the questioning and examination tasks should gear towards the promoting of information displayed graphically.

Theoretically, the finding of the study is able to contribute towards a better and in depth understanding of how students employ graphs as a strategy in problem solving. We anticipate that there are potentially some reasons that contribute to the students' reluctance to opt graphs such as the familiarity with the procedural processes and the skills of drawing graphs (Kool, McGuire, Rosen and Botvinick, 2010). Future research in the area would need to investigate students' knowledge and cognitive load (Uesaka & Manalo, 2011) and graphs or any other types of representations and properties such that the understanding could help the educational system in terms of the teaching method to promote the use of graphs in appropriate problem solution. There are various ways of how experts such as educators and mathematicians can use graphs to represent situations and relationships to assist students in the solving of problems. Their knowledge on graphs can be coordinated along two dimensions: the display characteristics of the graphs and the functions of the graphs (i.e. what are the graphs for or represent). Different types and functions of graphs are associated with different load of cognitive demands, it is therefore recommended that the variations be highlighted to seek students' skills and ability to sketch graphs and to read and interpret graphs efficiently and effectively.

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